CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST FOUR 2016
Year 12
Non Calculator Section
Chapters 6, 7, 8

Name________________________________

Time: 15 minutes
Total: 13 marks

1 [5 Marks]

a) Find the expression for $\frac{dy}{dx}$ given the relationship $e^{\cos(x)} + e^{\sin(y)} = e+1$ (3)

$-\sin x\, e^{\cos x} + \cos y\, e^{\sin y}\frac{dy}{dx} = 0$ ✓✓✓

$\frac{dy}{dx} = \frac{\sin x\, e^{\cos x}}{\cos y\, e^{\sin y}}$ ✓

b) Hence find $\frac{dy}{dx}$ at the point $x = 0$ (2)

when $x = 0$ $y = 0$. ✓

$\therefore \frac{dy}{dx} = 0$

2 **[8 Marks]**

a) Find the gradient of the tangent to the curve $xy^2 = 4 + 3yx^3, y > 0$ when $x = 1$ (4)

$$1y^2 + 2xy\frac{dy}{dx} = 9x^2y + 3x^3\frac{dy}{dx}$$

$$9x^2y - y^2 = (2xy - 3x^3)\frac{dy}{dx}$$, $x=1$, $y^2 = 4 + 3y$

$$y^2 - 3y - 4 = 0$$

$$(y-4)(y+1) = 0$$

$$\therefore y = 4$$

as $y > 0$

$$\frac{dy}{dx} = \frac{9x^2y - y^2}{2xy - 3x^3}$$

$$= \frac{9.1.4 - 4^2}{2.1.4 - 3.1^3}$$

$$= \frac{36 - 16}{8 - 3} = \frac{20}{5}$$

$$= 4$$

b) If $y = \sin(x^2)$, show that $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = 0$ (4)

$$\frac{dy}{dx} = 2x\cos(x^2)$$

$$\frac{d^2y}{dx^2} = 2\cos(x^2) + 2x \cdot 2x \cdot -\sin(x^2)$$

$$= 2\cos(x^2) - 4x^2\sin(x^2)$$

$$LHS = \cancel{2\cos(x^2)} - 4x^2\sin(x^2) - \frac{1}{\cancel{x}}\cancel{2x}\cancel{\cos(x^2)} + 4x^2y$$

$$= -4x^2\sin(x^2) + 4x^2(\sin(x^2))$$

$$= 0$$

$$= RHS$$

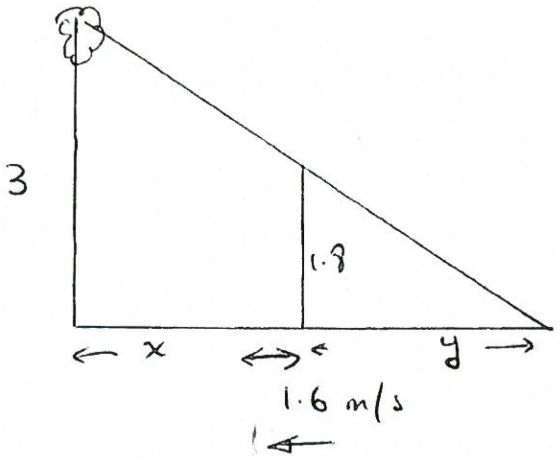
Name____

Time: 40 minutes
Total: 35 marks

3 [6 Marks]

A person of height 1.8 m is walking directly toward a light pole at night. The light is 3 m above the ground, and the person is walking at 1.6 m/s on level ground. At what rate is

a) the length of the shadow decreasing? (4)



Given

$\frac{dx}{dt} = -1.6$ m/s towards ✓

$\frac{3}{1.8} = \frac{x+y}{y}$ △'s similar ✓

$\Rightarrow 3y = 1.8x + 1.8y$

$y = 1.5x$

$\Rightarrow \frac{dy}{dx} = 1.5$ ✓

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= -1.5 \times 1.6$

$= -2.4$ m/s ✓

b) the tip of the shadow moving when the person is 4 m from the foot of the light pole? (2)

✓

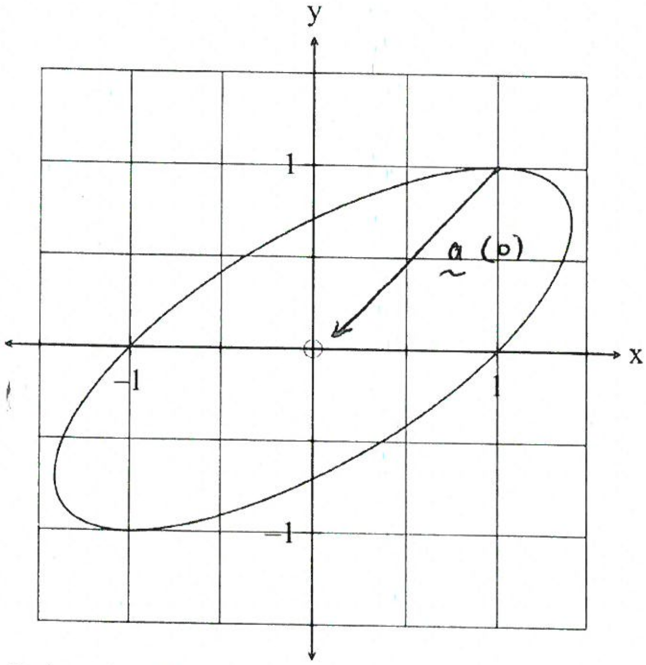
as $\frac{dy}{dt}$ is constant. then the rate for the shadow

is $-2.4 + -1.6 = -4$ m/s ✓

6

4 **[9 Marks]**

(a) The position vector of a particle travelling on an elliptical path, as shown on the graph below, is given by $r(t) = (sin(t) + cos(t))i + (cos(t))j$ for any time t.



(i) Find when the particle is at (-1, -1). (2)

$-1 = \sin(t) + \cos(t)$ and $-1 = \cos t$

$-1 = \sin t - 1$ $t = \pi$

$\Rightarrow \sin(t) = 0$ ✓ $\therefore \underline{t = \pi}$ ✓

(ii) Find the initial position of the particle. (1)

$t = 0$, position is $(1, 1)$

3

(iii) Find the velocity and acceleration of the particle at $t = 0$. (3)

$$\underset{\sim}{r}(t) = (\sin(t) + \cos(t))\underline{i} + \cos(t)\underline{j}$$

$$\underline{v}(t) = (\cos(t) - \sin(t))\underline{i} - \sin(t)\underline{j} \quad \checkmark$$

$$\underset{\sim}{a}(t) = (-\sin(t) - \cos(t))\underset{\sim}{i} - \cos(t)\underline{j} \quad \checkmark$$

$$\Rightarrow V(0) + \underline{i}$$

$$a(0) = -\underline{i} - \underline{j} \quad \checkmark$$

(iv) Plot the acceleration vector on the graph at $t = 0$. (2)

$\checkmark\checkmark$ see graph.

{ directn towards centre from (1, 1)

(v) Determine the values of t such that $\boldsymbol{a}(t) = -\boldsymbol{r}(t)$. (1)

$$r(t) = (\sin(t) + \cos(t))\underline{i} + \cos(t)\underline{j}$$

$$a(t) = (-\sin(t) - \cos(t))\underset{\sim}{i} - (\cos(t))\underline{j}$$

$$= -[(\sin(t) + \cos(t))\underline{i} + (\cos(t))\underline{j}]$$

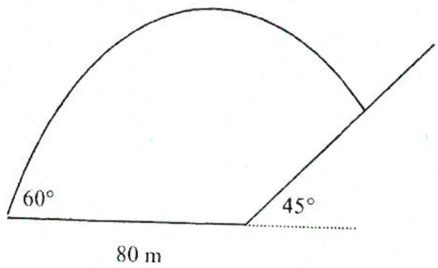
$$= -r(t)$$

hence true for all values of t.

6

5 **[14 marks]**

A golfer is playing a shot on the moon 80 metres from the edge of a hill, which has a slope of 45° - as shown in the diagram below. Assume gravity is 1.2 m / sec2 downwards, and that the position in which the ball is struck is the origin.



He hits the ball with a velocity of 12 m/sec at an angle of 60°.

a) Show why the velocity of the ball at any time t, seconds, is given by
$\mathbf{v}(t) = 6\mathbf{i} + (6\sqrt{3} - 1.2t)\mathbf{j}$

(4)

$\underset{\sim}{a}(t) = -1.2\underset{\sim}{j}$ ✓

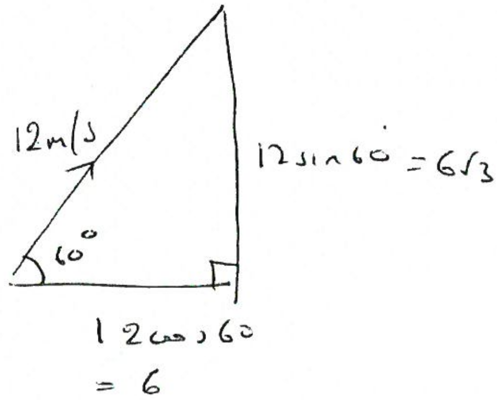
given that $V(t=0) = 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$ ✓

$v(t) = \int \underset{\sim}{a}(t)\,dt = -1.2t\underset{\sim}{j} + c$

so $t=0$ then $v(t) = 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$
$= c$ ✓

$\therefore V(t) = -1.2t\underset{\sim}{j} + 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$

$= 6\underset{\sim}{i} + (6\sqrt{3} - 1.2t)\underset{\sim}{j}$ ✓



b) Determine the <u>position</u> of the ball at any time t.

(2)

$r(t) = 6t\underset{\sim}{i} + (6\sqrt{3}t - 0.6t^2)\underset{\sim}{j} + c$

when $t=0$, $c=0$.

$\Rightarrow r(t) = 6t\underset{\sim}{i} + (6\sqrt{3}t - 0.6t^2)\underset{\sim}{j}$ ✓✓

6

c) Determine the height of the ball when minimum speed is attained. (3)

$$\text{Speed} = \sqrt{6^2 + (6\sqrt{3} - 1.2t)^2}$$

Solve quadratic min

ie min when $\underline{3t}$ $\therefore t = 8.66$ sec

$\therefore$ Height is the $\underline{j}$ comp $6\sqrt{3}(8.66) - 0.6(8.66)^2 = 44.999 \simeq \underline{50}$ m.

45 m

d) Determine the Cartesian equation for the relationship between y and x for the position vector of the ball by considering the parametric equations for the x and y components. (2)

$$x = 6t \quad \ldots (1)$$

$$y = 6\sqrt{3}t - 0.6t^2 \quad \ldots (2)$$

From (1) $t = \dfrac{x}{6} \quad \ldots (3)$

Sub (3) in (2)

$$y = \frac{\cancel{6}\sqrt{3}x}{\cancel{6}} - \frac{0.6x^2}{36}$$

$$= \sqrt{3}x - \frac{x^2}{60}$$

e) Hence, or otherwise, determine the height of the hill at the position that the ball hits the hill. (Hint: Define y in terms of x for the equation of the hill first!) (3)

$y_1 = \sqrt{3}x - \dfrac{x^2}{60}$, ball

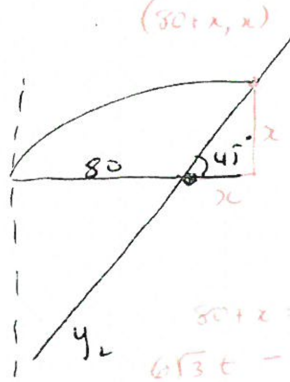
$y_2 = x - 80$

$y_1 = y_2$ = Graph & solve

$x = 94.64$

$y = \underline{14.64}$ m

Not much of a $\underline{\text{hill!}}$



$y = mx + c$

$= x + c$

$0 = 80 + c$

$c = -80$

$80 + x = 6t$

$6\sqrt{3}t - 0.6t^2 = x$

$x = 14.64$

Hill height is

14.64 m

8

6 [2 marks]

Solve the following system of linear equations where possible. If there is more than one solution, or no solution state why clearly. If there is one solution, find it.

$$x + y + z = 2$$
$$x - 2y + 3z = 8$$
$$2x - y + 4z = 10$$

$$\downarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -3 & -2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

None of planes are // or identical

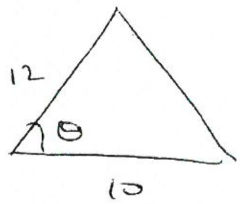
$\Rightarrow$ Planes intersect in a line. ∞ solutions

7 [4 marks]

A triangle's area is given by $A = \frac{ab}{2}\sin\theta$, where a and b are the lengths of two sides determining angle θ.

If the two sides of length 10cm and 12cm have an included angle θ increasing at 1^0 per minute, determine

a) $\frac{d\theta}{dt}$ in **radians** per minute exactly; (1)



$$\frac{d\theta}{dt} = \frac{\pi}{180}$$

1^0 / min

$\left(\frac{d\theta}{dt} = \frac{1}{360^0} \times 2\pi\right)$

b) A, in terms of θ only; (1)

$A = 60\sin\theta$

c) exactly how fast the area of the triangle is changing with respect to time when the included angle is 120^0. (2)

$$\frac{dA}{dt} = \frac{dA}{d\theta} \cdot \frac{d\theta}{dt}$$
$$= 60\cos\theta \cdot \frac{d\theta}{dt}$$
$$= 60\cos\theta \cdot \frac{d\theta}{dt}$$
$$= -\frac{15}{\pi} \text{ cm}^2/\text{min}$$

$-60 \times \left(\frac{1}{2}\right) \times \frac{2\pi}{360}$

$= -\frac{\pi}{6} \text{ cm}^2/\text{min}$

6